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Resonant Frequency Stability of the Dielectric Resonator on a Dielectric Substrate

TORU HIGASHI AND TOSHIHIKO MAKINO

***Abstract*—A simple approximate method for predicting the resonant frequencies of TE modes of dielectric resonators is developed. By using this method, an analytical expression is derived for the resonant frequency stability of the dielectric resonator on a dielectric substrate, and the effect of the substrate on the stability is studied. The result is useful when the high-frequency stability is required.**

I. Introduction

DIELECTRIC resonators exhibiting high Q factors and very low temperature dependence of the resonant frequency have been recently developed [1]–[3]. They promise to shrink the size and cost of waveguide cavities. Also, they are useful elements of MIC's, and have been applied for filters [4] and oscillators [5]–[7].

A dielectric resonator structure commonly used in practical MIC's is that composed of a cylindrical dielectric sample placed on a dielectric substrate, one side of which is metallized as a ground plane, and of a metal tuning screw placed above the dielectric resonator sample. On the calculation of such a dielectric resonator structure, several works have been reported [8], [9]. However, none of them has described the resonant frequency stability. The degree of effect of the factors affecting the resonant frequency stability must be considered when the high stability is required especially in a local oscillator application [7].

The purpose of the present paper is to derive the analytical formula for the resonant frequency stability of the TE mode dielectric resonator on a dielectric substrate, and to show how the substrate affects the frequency stability. The result is useful in determining the temperature coefficient of the dielectric resonator material to realize the high-frequency stability in a practical MIC application.

II. Approximate Resonant Frequency

The resonant structure under consideration is shown schematically in Fig. 1. D is the diameter, and L the height of the dielectric resonator. ϵ_1, ϵ_2, and ϵ_r are the relative

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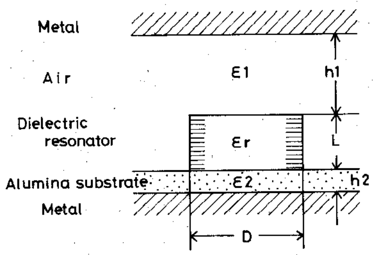

Fig. 1. The dielectric resonator structure.

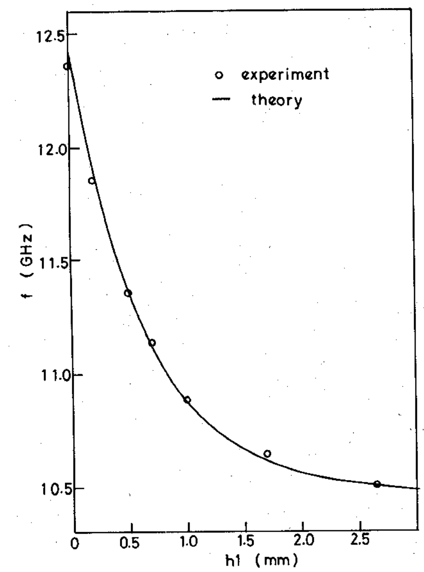

Fig. 2. The tuning of a cylindrical resonator placed in MIC, $h_2=0.62$ mm, $\epsilon_1=1.0$, $\epsilon_2=9.4$, $\epsilon_r=36.4$, $D=5.66$ mm, $L=2.26$ mm.

dielectric constants of the air, the substrate, and the dielectric resonator, respectively. h_1 is the distance between the dielectric resonator and the upper metal wall, and h_2 the thickness of the substrate.

The resonant conditions for TE modes are given by [8], [9]

$$x_\rho^2+x_z^2=\epsilon_r x_0^2 \tag{1}$$

$$x_\rho^2-x_1^2=\epsilon_1 x_0^2 \tag{2}$$

$$x_\rho^2-x_2^2=\epsilon_2 x_0^2 \tag{3}$$

$$x_\rho^2+x_\kappa^2=(\epsilon_r-1)x_0^2 \tag{4}$$

$$x_\rho J_0(x_\rho)/J_1(x_\rho)=-x_\kappa K_0(x_\kappa)/K_1(x_\kappa) \tag{5}$$

$$\xi=\left[\tan^{-1}(p_1/x_z)+\tan^{-1}(p_2/x_z)\right]/2x_z \tag{6}$$

$$\xi=L/D \tag{7}$$

$$x_0=\pi D f_0/c \tag{8}$$

$$p_i=x_i\coth(2x_ih_i/D), \qquad i=1,2 \tag{9}$$

where f_0 is the resonant frequency, c the light velocity, J_n is the Bessel function of the first kind of nth order, and K_n the modified Hankel function of nth order.

From (4), and (5), x_ρ is obtained approximately as

$$x_\rho=0.951\rho_{01}+0.222\sqrt{(\epsilon_r-1)x_0^2-0.951\rho_{01}^2} \tag{10}$$

where $\rho_{01}=2.405$ (the first root of the equation $J_0(x)=0$). This approximation is good for the range $\sqrt{\epsilon_r-1}\,x_0<4$, which holds in practical cases. If we use this approximation, we can easily obtain the relation between x_0 and ξ from (1) through (3) and (6), and express the resonant frequency stability in an analytical form (see Section III).

The comparison between the resonant frequency of the $TE_{01\delta}$ mode computed by this method and the experimental result is given in Fig. 2. The theoretical values agree well with the experimental.

III. Resonant Frequency Stability

If we denote the small change of quantity A by ΔA, we can generally express the resonant frequency stability $\Delta f_0/f_0$ as

$$\frac{\Delta f_0}{f_0}=C_D\frac{\Delta D}{D}+C_L\frac{\Delta L}{L}+C_{h1}\frac{\Delta h_1}{h_1}+C_{h2}\frac{\Delta h_2}{h_2}+C_{\epsilon 1}\frac{\Delta\epsilon_1}{\epsilon_1}+C_{\epsilon 2}\frac{\Delta\epsilon_2}{\epsilon_2}+C_{\epsilon r}\frac{\Delta\epsilon_r}{\epsilon_r}. \tag{11}$$

From (1) through (10), the above coefficients (C_D, etc.,) are given by the following expression (see Appendix):

$$C_D=-\frac{1+M+A_1\gamma_1+A_2\gamma_2}{M} \tag{12}$$

$$C_L=\frac{1}{M} \tag{13}$$

$$C_{h1}=\frac{A_1\gamma_1}{M} \tag{14}$$

$$C_{h2}=\frac{A_2\gamma_2}{M} \tag{15}$$

$$C_{\epsilon 1}=-\frac{A_1(1-\gamma_1)\Omega_1}{M} \tag{16}$$

$$C_{\epsilon 2}=-\frac{A_2(1-\gamma_2)\Omega_2}{M} \tag{17}$$

$$C_{\epsilon r}=-\frac{A_1\theta_1(1-\gamma_1)+A_2\theta_2(1-\gamma_2)-\Gamma_1(1+A_1+A_2)}{M} \tag{18}$$

where

$$M=-\Gamma_2(1+A_1+A_2)+A_1\Delta_1(1-\gamma_1)+A_2\Delta_2(1-\gamma_2)$$

$$\Gamma_1=\frac{\epsilon_r x_0^2}{2x_z^2}\left[1-\frac{b^2x_\rho}{x_\rho-a}\right]$$

$$\Gamma_2=\frac{x_0^2}{x_z^2}\left[\epsilon_r-\frac{b^2x_\rho(\epsilon_r-1)}{x_\rho-a}\right]$$

$$\Delta_i=\frac{x_0^2}{x_i^2}\left[\frac{b^2x_\rho(\epsilon_r-1)}{x_\rho-a}-\epsilon_i\right]$$

$$\theta_i=\frac{\epsilon_r b^2x_0^2x_\rho}{2x_i^2(x_\rho-a)}$$

$$\Omega_i=-\frac{x_0^2\epsilon_i}{2x_i^2}. \tag{19}$$

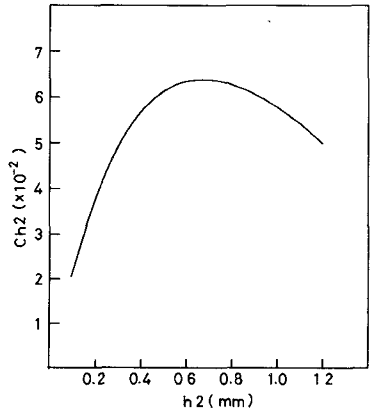

Fig. 3. The stability coefficient C_{h2} versus the thickness of substrate h_2. $h_1=2.0$ mm, $\epsilon_1=1.0$, $\epsilon_2=9.4$, $\epsilon_r=35.0$, $D=5.66$ mm, $L=2.26$ mm

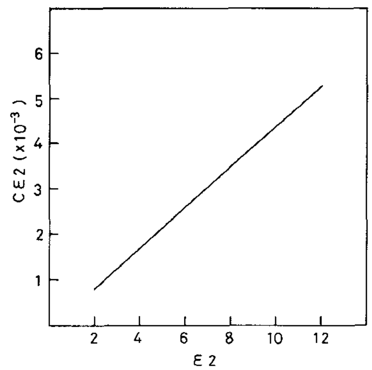

Fig. 4. The stability coefficient $C_{\epsilon 2}$ versus the dielectric constant of substrate ϵ_2. $h_1=2.0$ mm, $h_2=0.62$ mm, $\epsilon_1=1.0$, $\epsilon_r=35.0$, $D=5.66$ mm, $L=2.26$ mm.

The computed values of the coefficients are shown in Table I in the case when $D=5.66$ mm, $L=2.26$ mm, $\epsilon_r=35$, $h_1=2.0$ mm, and $\epsilon_1=1.0$.

When the resonator material is isotropic, we can assume that $\Delta D/D=\Delta L/L$. Then

$$C_D \Delta D/D+C_L \Delta L/L=(C_D+C_L)\Delta L/L. \tag{20}$$

From Table I, we obtain that $C_D+C_L \cong -0.9$. This indicates that the main factor affecting the resonant frequency stability is the change rate of dimension of a dielectric resonator. The second factor is the change rate of resonator dielectric constant ($C_{\epsilon r} \cong -0.5$). For the dielectric resonator both sides of which are terminated by metals, the equation

$$\frac{\Delta f_0}{f_0}=\frac{\Delta L}{L}+\frac{1}{2}\frac{\Delta \epsilon_r}{\epsilon_r} \tag{21}$$

holds in a good approximation. While it is seen that the above equation is a good approximation also in the present case, the other factors must be considered when the high-frequency stability is required. For example, the alumina substrate reported in [10] has the values given by

$$(1/\Delta T)\Delta h_2/h_2=7.2 \text{ ppm/°C}$$
$$(1/\Delta T)\Delta \epsilon_2/\epsilon_2=110 \text{ ppm/°C}$$

TABLE I

	f [GHz]	-CD	-CL	-CEr	-CE2	-Ch1	-Ch2	-CE1
E2=2.0, h2=0.62mm	10.80	0.567	0.363	0.493	0.811×10^{-3}	0.132×10^{-1}	0.574×10^{-1}	0.199×10^{-2}
E2=8.0, h2=0.62mm	10.78	0.568	0.359	0.490	0.344×10^{-3}	0.131×10^{-1}	0.625×10^{-1}	0.197×10^{-2}
E2=9.4, h2=0.40mm	11.08	0.540	0.391	0.492	0.155×10^{-3}	0.133×10^{-1}	0.564×10^{-1}	0.214×10^{-2}
E2=9.4 h2=0.80mm	10.58	0.586	0.338	0.487	0.867×10^{-3}	0.130×10^{-1}	0.627×10^{-1}	0.186×10^{-2}

h1=2.0mm, L=2.26mm, D=5.66mm, E1=1.0, Er=35

where ΔT denotes the small change of temperature. Using the above values and the values of C_{h2} and $C_{\epsilon 2}$ for the case when $h_2=0.8$ mm and $\epsilon_2=9.4$ (see Table I), we obtain

$$C_{h2}(1/\Delta T)\Delta h_2/h_2=-0.45 \text{ ppm/°C}$$

and

$$C_{\epsilon 2}(1/\Delta T)\Delta \epsilon_2/\epsilon_2=-0.73 \text{ ppm/°C}.$$

Therefore, the effect of the substrate cannot be neglected when the high frequency stability is required. Figs. 3 and 4 show C_{h2} versus h_2 and $C_{\epsilon 2}$ versus ϵ_2, respectively. It should be noted that the resonant frequency stability changes if the thickness or the material of the substrate is changed even though the dielectric resonator sample is not changed.

To tune the resonant frequency of the dielectric resona-

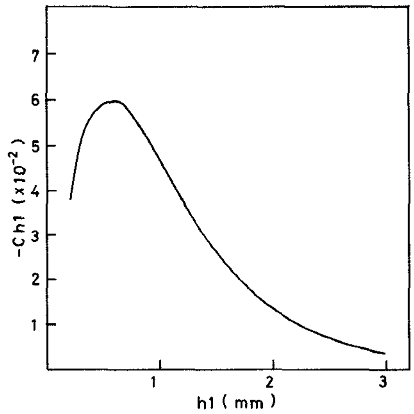

Fig. 5. The stability coefficient C_{h1} versus the distance between the dielectric resonator and the upper metal wall h_1. $h_2=0.62$ mm, $\epsilon_1=1.0$, $\epsilon_2=9.4$, $\epsilon_r=36.4$, $D=5.66$ mm, $L=2.26$ mm.

tor of TE mode, h_1 is usually changed as is shown in Fig. 2. In this case, it should be noted that the change of h_1 results in the change of frequency stability. Fig. 5 shows C_{h1} versus h_1.

IV. Conclusion

A simple approximate method for predicting the resonant frequencies of TE modes of the dielectric resonator on a dielectric substrate has been developed, and it has provided the result in close agreement with the experiment. By using this method, an analytical expression for the resonant frequency stability has been derived, and the effects of the substrate and the tuning screw on the frequency stability have been studied.

Appendix

In the resonant structure of Fig. 1, the resonant conditions are

$$\xi = L/D \tag{A.1}$$

$$x_0 = \frac{\pi}{c} f_0 D \tag{A.2}$$

$$x_z = \sqrt{\epsilon_r x_0^2 - x_\rho^2} \tag{A.3}$$

$$x_i = \sqrt{x_\rho^2 - \epsilon_i x_0^2}, \qquad i=1,2 \tag{A.4}$$

$$x_\rho = a + b\sqrt{(\epsilon_r - 1)x_0^2 - a\rho_{01}}, \quad a=0.951\rho_{01}, \quad b=0.222, \quad \rho_{01}=2.405 \tag{A.5}$$

$$p_i = x_i \coth \zeta_i \tag{A.6}$$

$$\zeta_i = \frac{2x_i h_i}{D} \tag{A.7}$$

and

$$\xi = \frac{1}{2x_z}\left[\tan^{-1}\left(\frac{p_1}{x_z}\right) + \tan^{-1}\left(\frac{p_2}{x_z}\right)\right]. \tag{A.8}$$

If we denote the small change of quantity A by ΔA and write

$$\delta A = \Delta A / A \tag{A.9}$$

we obtain from (A.1) through (A.5)

$$\delta\xi = \delta L - \delta D \tag{A.10}$$

$$\delta x_0 = \delta D + \delta f_0 \tag{A.11}$$

$$\delta x_z = \Gamma_1 \delta\epsilon_r + \Gamma_2 \delta x_0 \tag{A.12}$$

$$\delta x_i = \Delta_i \delta x_0 + \theta_i \delta\epsilon_r + \Omega_i \delta\epsilon_i, \qquad i=1,2 \tag{A.13}$$

where

$$\Gamma_1 = \frac{\epsilon_r x_0^2}{2x_z^2}\left[1 - \frac{b^2 x_\rho}{x_\rho - a}\right] \tag{A.14}$$

$$\Gamma_2 = \frac{x_0^2}{x_z^2}\left[\epsilon_r - \frac{b^2 x_\rho(\epsilon_r - 1)}{x_\rho - a}\right] \tag{A.15}$$

$$\Delta_i = \frac{x_0^2}{x_i^2}\left[\frac{b^2 x_\rho(\epsilon_r - 1)}{x_\rho - a} - \epsilon_i\right] \tag{A.16}$$

$$\theta_i = \frac{\epsilon_r b^2 x_0^2 x_\rho}{2x_i^2(x_\rho - a)} \tag{A.17}$$

$$\Omega_i = -\frac{x_0^2 \epsilon_i}{2x_i^2}. \tag{A.18}$$

It follows from (A.8) that

$$\delta\xi = -(1 + A_1 + A_2)\delta x_z + A_1 \delta p_1 + A_2 \delta p_2 \tag{A.19}$$

where

$$A_i = \frac{1}{2x_z^2 \xi} \frac{p_i}{1 + (p_i/x_z)^2}. \tag{A.20}$$

On the other hand, we obtain from (A.6)

$$\delta p_i = \delta x_i - c_i\left(\frac{\Delta\zeta_i}{x_i}\right) \tag{A.21}$$

where

$$c_i = (p_i^2 - x_i^2)/p_i. \tag{A.22}$$

From (A.7), we obtain

$$\frac{\Delta\zeta_i}{x_i} = \frac{2h_i}{D}\delta x_i + \frac{2h_i}{D}\delta h_i - \frac{2h_i}{D}\delta D. \tag{A.23}$$

Substitution of (A.23) into (A.21) gives

$$\delta p_i = (1-\gamma_i)\,\delta x_i - \gamma_i \,\delta h_i + \gamma_i \,\delta D \tag{A.24}$$

where

$$\gamma_i = \frac{2h_i c_i}{D}. \tag{A.25}$$

Substituting (A.10)–(A.13) and (A.24) into (A.19) we obtain

$$\delta f_0 = C_D \,\delta D + C_L \,\delta L + C_{h1} \delta h_1 + C_{h2} \,\delta h_2 + C_{\epsilon 1} \delta \epsilon_1 + C_{\epsilon 2} \,\delta \epsilon_2 + C_{\epsilon r} \delta \epsilon_r \tag{A.26}$$

where

$$C_D = -\frac{1+M+A_1\gamma_1 + A_2\gamma_2}{M} \tag{A.27}$$

$$C_L = \frac{1}{M} \tag{A.28}$$

$$C_{h1} = \frac{A_1\gamma_1}{M} \tag{A.29}$$

$$C_{h2} = \frac{A_2\gamma_2}{M} \tag{A.30}$$

$$C_{\epsilon 1} = -\frac{A_1(1-\gamma_1)\Omega_1}{M} \tag{A.31}$$

$$C_{\epsilon 2} = -\frac{A_2(1-\gamma_2)\Omega_2}{M} \tag{A.32}$$

$$C_{\epsilon r} = -\frac{A_1\theta_1(1-\gamma_1) + A_2\theta_2(1-\gamma_2) - \Gamma_1(1+A_1+A_2)}{M} \tag{A.33}$$

$$M = -\Gamma_2(1+A_1+A_2) + A_1\Delta_1(1-\gamma_1) + A_2\,\Delta_2(1-\gamma_2). \tag{A.34}$$

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